Comment on "'t Hooft vertices, partial quenching, and rooted staggered QCD"

Michael Creutz

Physics Department,

Brookhaven National Laboratory

Upton, NY 11973, USA

Abstract

A recent criticism of the proof of the failure of the rooting procedure with staggered fermions is shown to be incorrect.

PACS numbers: 11.15.Ha,11.30.Rd,12.38.Aw

In a recent paper [1] Bernard, Golterman, Shamir and Sharp challenge the proof developed in [2–4] showing that non-perturbative effects are incorrectly treated in the rooting formalism popular for reducing the number of fermion species in the staggered formalism for dynamical quarks. Here I discuss how this challenge is based on a serious misunderstanding of the chiral behavior of staggered quarks.

The problem appears already in the introduction of [1] where the authors attempt to define something they call the "rooted continuum theory" (RCT). They define this in two inequivalent ways. First they consider the continuum limit of staggered fermions treated with the rooting trick. But then towards the end of the introduction they say the RCT is the rooted theory obtained from the four copies of a chirally invariant formulation, such as with the overlap operator [5]. The whole point of the discussion in Ref. [2–4] is that these two theories display different non-perturbative effects. Only the latter form generates the correct one flavor theory. Confusing these theories is equivalent to assuming that rooting is correct. Throughout the rest of the paper they make no distinction between these definitions, just referring to the RCT as the physical one flavor theory. If the RCT operator is chosen from rooting four equivalent copies of a properly defined chiral fermion theory, then the remaining discussion in Ref. [1] is simply a verification of the trivial reduction to the one flavor case.

As discussed in some detail in Ref. [3], the four tastes in unrooted staggered fermions are not equivalent, but appear as parity partners. In particular, these behave differently in a background gauge configurations that are not parity invariant. Integration over the gauge fields restores overall parity and the fermions become physically equivalent, but it is incorrect to bring the root of the fermion determinant inside the gauge field integration. The problem becomes most serious when the gauge fields have non-trivial topology [6], in which case there are two left handed low lying eigenvectors of the staggered operator and two corresponding right handed modes. In contrast, with four flavors of equivalently defined overlap fermions, the tastes are, by construction, completely equivalent.

The introduction to Ref. [1] also propagates the mistaken illusion that it is only taste breaking and mixing that can cause problems. The serious problem with rooting is not taste breaking but the strong coupling between the tastes induced by non-perturbative effects. This coupling makes it impossible to construct physical observables involving only one taste at a time.

That taste breaking and the strong coupling between tastes are independent issues has recently been addressed in Ref. [7], where a two taste model for rooting is constructed with taste mixing explicitly removed. The arguments of Refs. [2–4] still apply, and rooting fails in this model also because the two tastes involved are physically inequivalent. The model is similar to Wilson fermions but considers one taste with an effective the strong interaction CP angle theta of $\pi/2$ and the second of $-\pi/2$, as discussed in Ref. [8]. This rotation allows a residual chiral symmetry to survive at finite lattice spacing, but does not commute with the rooting process. Considered as a two flavor theory, these phases cancel, but on rooting one is working with a mixture of two inequivalent one-flavor theories.

The authors of Ref. [1] proceed to rewrite the partition function for their theory in terms of a partially quenched theory with three ghost fields. Again they do not distinguish the RCT Dirac operator used and assume they can use the same one for each field, including the ghosts. Actually, Ref. [3] also raises the possibility of using ghost fields to reduce the flavor content of unrooted staggered quarks, but with the important proviso that the ghosts must be formulated with a chiral operator, such as the overlap, to properly cancel the inequivalent tastes.

Unfortunately, the authors of Ref. [1] have seriously misinterpreted the point of Ref. [3] on the effect of an instanton anti-instanton pair on the correlation between two pseudoscalar gluonic operators at a non-trivial separation. In particular, the issue is not the space time dependence of this correlator. That is controlled by the physical size of the topological defect, which in turn is of order the QCD scale. The real question is the mass dependence of these effects. With the theory defined by rooting the staggered action, an unphysical singularity appears as the mass goes to zero. For the rooted theory defined with a proper chiral foundation, this singularity is removed by Pauli statistics. This does not apply for the rooted staggered action where the four tastes are independent fermions, all appearing in the fermion propagator independent of rooting.

In summary, Ref. [1] confuses two different rooted theories, one of which is correct and the other not. The distinction is a strong inter-taste coupling that survives in the continuum limit and does not allow the effects of a single taste to be isolated. The successes of past simulations do suggest that these effects can be small for some observables, but it is incorrect to claim that they go away in the continuum limit.

The lattice provides a powerful tool for calculating quantum field theories in a way that goes beyond perturbation theory. The argument that staggered simulations are much faster than alternative approaches has recently become moot [9]. There really is no excuse for doing lattice gauge calculations incorrectly.

Acknowledgements

This manuscript has been authored under contract number DE-AC02-98CH10886 with the U.S. Department of Energy. Accordingly, the U.S. Government retains a non-exclusive, royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so, for U.S. Government purposes.

[1] C. Bernard, M. Golterman, Y. Shamir and S. R. Sharpe, Phys. Rev. D (in press), arXiv:0711.0696 [hep-lat].

[2] M. Creutz, Phys. Lett. B **649**, 230 (2007) [arXiv:hep-lat/0701018].

[3] M. Creutz, PoS **LATTICE2007**, 007 (2007) [arXiv:0708.1295 [hep-lat]].

[4] M. Creutz, Annals of Physics (in press), arXiv:0711.2640 [hep-ph].

[5] H. Neuberger, Phys. Lett. B417 (1998) 141; H. Neuberger, Phys. Lett. B427 (1998) 353; R. Narayanan and H. Neuberger, Phys. Lett. B302 (1993) 62; Phys. Rev. Lett. 71 (1993) 3251; Nucl. Phys. B412 (1994) 574; Nucl. Phys. B443 (1995) 305.

[6] G. 't Hooft, Phys. Rev. D **14**, 3432 (1976) [Erratum-ibid. D **18**, 2199 (1978)].

[7] D. H. Adams, Phys. Rev. D (in press), arXiv:0802.3029 [hep-lat].

[8] E. Seiler and I. O. Stamatescu, Phys. Rev. D **25**, 2177 (1982) [Erratum-ibid. D **26**, 534 (1982)].

[9] L. Del Debbio, L. Giusti, M. Luscher, R. Petronzio and N. Tantalo, JHEP **0702**, 056 (2007) [arXiv:hep-lat/0610059].